KLUB NAUKOWY FENIX

Problems 1

25 September, 2020

For reference use: W. Krysicki, L. Wlodarski, Analiza matematyczna w zadaniach, PWN 1988.

1) Find determinant of the matrix:

a) $\begin{bmatrix} 3 & 7 \\ -2 & 9 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 3 & 3 \\ -1 & 2 & -5 \\ 4 & 5 & 13 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 2 & -1 & -2 & 3 \\ 1 & 4 & -4 & 0 & 1 \\ -7 & 3 & 6 & -12 & 0 \\ 5 & 0 & -1 & 2 & 2 \\ 1 & 13 & 5 & -2 & -9 \end{bmatrix}$

2) Find solutions to set of equations:

a) $\begin{aligned} 5x + 3y - z &= 3 \\ 2x + y - z &= 1 \\ 3x - 2y + 2z &= -4 \\ x - y + 2z &= -2 \end{aligned}$, b) $\begin{aligned} x + 2y + 3z - 2t + u &= 4 \\ 3x + 6y + 5z - 4t + 3u &= 5 \\ x + 2y + 7z - 4t + u &= 11 \\ 2x + 4y + 2z - 3t + 3u &= 6 \end{aligned}$

3) Solve:

a) $|\sin t| = |\cos t|$ b) $\cos t = -1$ c) $\sin t = \sqrt{2} + \cos t$

4) Simplify:

a) $\sin^4 t + 2\sin^2 t \cos^2 t + \cos^4 t$ b) $\sin^4 t - \cos^4 t + \cos^2 t$ c) $\dfrac{1}{1 + \tan^2 t} + \dfrac{1}{1 + \cot^2 t}$

5) Express using $\sin t$ and $\cos t$ ($n \in \mathbb{N}$):

a) $\sin 2t$ b) $\cos 2t$ c) $\sin 4t$ d) $\cos 4t$ e) $\sin 5t$ f) $\cos 5t$ g) $\sin nt$ h) $\cos nt$

6) A bead is free to move along a wire bend in the shape of a parabola $y = \alpha x^2$. System is placed in gravitational field $\overrightarrow{g} = -g\hat{y}$ and wire is rotated around the y axis with angular velocity ω:
a) Find equilibrium positions for the bead,
b) Find force acting on the bead that is moved slightly out of balance.

7) A bead is moving with velocity v along a straight wire that rotates in (x, y) plane around the z axis with angular velocity ω. Find the force acting on the bead from the wire.