Problems 4

16 October, 2020

Taylor expansions

1. Find linear approximations to following functions near $x = 0$

   (α) $\sin(x)$
   (β) $\cos(x)$
   (γ) $\sin(\cos(x))$
   (δ) $\tan(x)$
   (ε) $\log(1 + x)$
   (ζ) $1/(1 + x)$
   (η) $\exp(x)$
   (θ) $\exp(x^2)$
   (ι) x^3
   (κ) $(x - 1)^{12}$

2. Find linear approximation to function $\sin(\tan(\log(1 + x)))$ near $x = 0$.

3. Find linear approximation to function $\sin((x + 1)(x + 2)$ near $x = -2$

First order ODEs

1. Find solution to the following differential equations for the function $y'(x)$

   (α) $y'(x) = 1$
   (β) $y'(x) = \sin(x)$
   (γ) $y'(x) = \cos(x)$
   (δ) $y'(x) = 2x$
   (ε) $y'(x) = \exp(x)$
   (ζ) $y'(x) = 1/x$
   (η) $y'(x) = y(x)$
   (θ) $y'(x) = -y(x)$
   (ι) $y'(x) = 5y(x)$

2. Sketch solutions to equation by drawing tangents at multiple different values of (x, y).

$$y'(x) = xy(x)$$

   Solve the equation to verify shape of your initial sketches.

3. Find solutions to the following initial and boundary value problems

   (α) $y'(x) = 1, \quad y(0) = 1$
   (β) $y'(x) = \sin(x), \quad y(0) = 0$
   (γ) $y'(x) = \cos(x), \quad y(0) = 1$
   (δ) $y'(x) = 2x, \quad 3y(0) = 2y(1)$
   (ε) $y'(x) = \exp(x), \quad y(0) = 0$
   (ζ) $y'(x) = 1/x, \quad y(1) = 2$
   (η) $y'(x) = y(x), \quad y(0) = 3$
   (θ) $y'(x) = -y(x), \quad y(0) = 3$
   (ι) $y'(x) = 5y(x), \quad y(0) = -1$

Some physics problems

1. In carbon dating proportion of C^{14} carbon to C^{12} carbon isotopes is used to determine for how long a given was unable to exchange carbon with the atmosphere. Half-life of C^{14} isotope is 5730 years. Assume you have an access to a device that is able to measure C^{14} proportion with 1 percentage point accuracy. What is the upper and lower bound of the age of the sample that you obtain? Now you could swap your device for a different one such that it measures this proportion with 10% relative accuracy. Would you take the offer? Does this depend on the sample age?

2. For very small objects in fluid Stokes drag is a good approximation of the force that fluid exerts on the particles. Consider a virus of size 100nm and weight 40000 kDa in water. Write down equation for dependence of velocity on time for such a virus. What is the characteristic timescale under which it decays? How long does it take to dissipate an energy of typical thermal fluctuation of k_bT at room temperature?