

Problems 10

27 November, 2020

Partial derivatives

1. Find the partial derivative with respect to x , y and z (where applicable) of the function:

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| (a) x^2y | (e) $\sin(x^2) \sqrt{\tanh y} - \exp(\sin z) \sin^2 y$ |
| (b) $\sqrt{x^2 + y^2 - 2xy \cos \alpha}$ | (f) $x^5y^{10} - x^3 \sin z + y^2 \exp z$ |
| (c) $x\sqrt{y} + \frac{y}{\sqrt{x}}$ | (g) $(\log x)^{\sin y}$ |
| (d) $(1 + xy)^y$ | (h) $2x^3y - 5x^2y^3 + x \cos 2y$ |

2. Show that function $f = \sqrt{x^2 + y^2 + z^2}$ solves the equation $\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + \left(\frac{\partial^2 u}{\partial y^2}\right)^2 + \left(\frac{\partial^2 u}{\partial z^2}\right)^2 = 1$.

3. Knowing that $x(t) = \sin t$ and $y(t) = t^2 \exp(-t)$ find the total derivative df/dt of the function:
 $f(x, y, t) = x^2t + y - 2t$.

Linear Differential Equations

1. Find the stationary solution for the $T(x, t)$ to the equation:

$$\frac{\partial^2 T}{\partial x^2} - \lambda \frac{\partial T}{\partial t} = 0 \quad (1)$$

assuming boundary conditions: $T(0, t) = T_1$, $T(L, t) = T_2$.

2. Knowing that $y(x, t) = Y(x) \cos \omega t$ and $x \in (0, L)$ solve the equation:

$$\frac{\partial^4 y}{\partial x^4} + \alpha \frac{\partial^2 y}{\partial t^2} = 0, \quad (2)$$

assuming boundary conditions: $Y(0) = 0$, $Y'(0) = 0$, $Y''(L) = 0$, $Y'''(L) = 0$ find an equation for ω . Using computer solver, or other method, find values of ω corresponding to five lowest modes of vibration.

Physics problems

- A distortion in the string given by the function $f(x - ct)$ approaches a fixed end of the string. Find the function $g(x + ct)$ of the reflected wave.
- A distortion given by the function $f(x - ct)$ approaches an end of the string that is attached to a pole such that the string can move freely along it. Find the function $g(x + ct)$ of the reflected wave.
- Find shape and frequency of normal modes of vibrations of a string of length L , stretched with tension T and with linear density ρ that is:
 - Fixed at both ends.
 - Fixed at one end and attached to a pole at the other end such that it can move freely along the pole.
 - Attached to poles at both ends such that it can move freely along the poles.