

Problems 11

4 December, 2020

The ∇

1. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be constant vectors. Show that:

$$(\alpha) \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0} \quad (\delta) \nabla \times (\mathbf{a} \times \mathbf{F}) = \mathbf{a}(\nabla \cdot \mathbf{F}) - (\mathbf{a} \cdot \nabla)\mathbf{F}$$

$$(\beta) \nabla(\mathbf{r} \cdot \mathbf{a}) = \mathbf{a}$$

$$(\varepsilon) \nabla \times (\mathbf{a} \times \mathbf{r}) = 2\mathbf{a}$$

$$(\gamma) (\mathbf{a} \cdot \nabla)\mathbf{r} = \mathbf{a}$$

$$(\zeta) \nabla(|\mathbf{r} - \mathbf{a}|)^{-1} = -\frac{\mathbf{r} - \mathbf{a}}{|\mathbf{r} - \mathbf{a}|^3}$$

2. Let $r = |\mathbf{x}|$ and \mathbf{a}, \mathbf{b} constant vectors. Find divergence and curl of the following:

$$(\alpha) r\mathbf{x}$$

$$(\gamma) \mathbf{a} \times \mathbf{x}$$

$$(\beta) \mathbf{a}(\mathbf{x} \cdot \mathbf{b})$$

$$(\delta) \mathbf{x}/r^3$$

Physics problems

1. Consider 3D space with current density $\mathbf{J} = C\mathbf{x}\exp(-at|\mathbf{x}|^2)$ with C, a constants. Find time dependent charge density knowing that $\rho = [f(\mathbf{x}) + tg(\mathbf{x})]\exp(-at|\mathbf{x}|^2)$ for some functions f, g .
2. Roughly draw electric field lines and equipotentials in a system of (i) a single charge, (ii) two identical charges separated by a distance (iii) charge q and charge $-q$ separated by a distance.