

Problems 1

24 September, 2021

For reference use: W. Kryszicki, L. Włodarski, Analiza matematyczna w zadaniach, PWN 1988.

1) Find determinant of the matrix:

$$\text{a) } \begin{bmatrix} 3 & 7 \\ -2 & 9 \end{bmatrix} \quad \text{b) } \begin{bmatrix} 2 & 3 & 3 \\ -1 & 2 & -5 \\ 4 & 5 & 13 \end{bmatrix} \quad \text{c) } \begin{bmatrix} 3 & 2 & -1 & -2 & 3 \\ 1 & 4 & -4 & 0 & 1 \\ -7 & 3 & 6 & -12 & 0 \\ 5 & 0 & -1 & 2 & 2 \\ 1 & 13 & 5 & -2 & -9 \end{bmatrix}$$

2) Find solutions to the set of equations:

$$\text{a) } \begin{cases} 5x + 3y - z = 3 \\ 2x + y - z = 1 \\ 3x - 2y + 2z = -4 \\ x - y + 2z = -2 \end{cases}, \quad \text{b) } \begin{cases} x + 2y + 3z - 2t + u = 4 \\ 3x + 6y + 5z - 4t + 3u = 5 \\ x + 2y + 7z - 4t + u = 11 \\ 2x + 4y + 2z - 3t + 3u = 6 \end{cases}$$

3) Solve:

$$\text{a) } |\sin t| = |\cos t| \quad \text{b) } \cos t = -1 \quad \text{c) } \sin t = \sqrt{2} + \cos t$$

4) Simplify:

$$\text{a) } \sin^4 t + 2 \sin^2 t \cos^2 t + \cos^4 t \quad \text{b) } \sin^4 t - \cos^4 t + \cos^2 t \quad \text{c) } \frac{1}{1 + \tan^2 t} + \frac{1}{1 + \cot^2 t}$$

5) Express using $\sin t$ and $\cos t$ ($n \in \mathbb{N}$):

$$\text{a) } \sin 2t \quad \text{b) } \cos 2t \quad \text{c) } \sin 4t \quad \text{d) } \cos 4t \quad \text{e) } \sin 5t \quad \text{f) } \cos 5t \quad \text{g) } \sin nt \quad \text{h) } \cos nt$$

6) A bead is free to move along a wire bend in the shape of a parabola $y = \alpha x^2$. System is placed in gravitational field $\vec{g} = -g\hat{y}$ and wire is rotated around the y axis with angular velocity ω :

- a) Find equilibrium positions for the bead,
- b) Find force acting on the bead that is moved slightly out of balance.

7) A bead is moving with velocity v along a straight wire that rotates in (x, y) plane around the z axis with angular velocity ω . Find the force acting on the bead from the wire.