KLUB NAUKOWY FENIX

Problems 2

1 October, 2021

For reference use: "Introduction to electrodynamics" David J. Griffiths, ISBN-13: 978-0-321-85656-2

1) Find Euclidean length of following vectors:

(a) $[1, 1, 1]$

(b) $[0, 0, 1]$

(c) $[\sin(x), \cos(x), 0]$

(d) $[\sin(x), \cos(x)\sin(y), \cos(x)\cos(y)]$

2) Find sum, scalar product and vector product of vectors:

(a) $\boldsymbol{A} = [1, 2, 0], \boldsymbol{B} = [5, 4, 0]$

(b) $\boldsymbol{A} = [0, 2, 4], \boldsymbol{B} = [4, 0, 0]$

(c) $\boldsymbol{A} = [3, 3, 0], \boldsymbol{B} = [-2, 1, 0]$

(d) $\boldsymbol{A} = [2, 4, 0], \boldsymbol{B} = [3, 6, 0]$

3) Determine the length of each vector and the angle between them:

(a) $\boldsymbol{A} = [1, 2], \boldsymbol{B} = [2, 5]$

(b) $\boldsymbol{A} = [0, 2, 4], \boldsymbol{B} = [3, 2, 0]$

4) Determine if following relation is true:

$$(\boldsymbol{A} \times \boldsymbol{B}) \times \boldsymbol{C} \stackrel{?}{=} \boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C})$$

If true, provide a proof; if false, provide a counterexample.

5) Show that:

(a) $(\boldsymbol{A} \times \boldsymbol{B}) \times \boldsymbol{C} = \boldsymbol{B}\,(\boldsymbol{A} \cdot \boldsymbol{C}) - \boldsymbol{C}\,(\boldsymbol{A} \cdot \boldsymbol{B})$

(b) $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) + \boldsymbol{B} \times (\boldsymbol{C} \times \boldsymbol{A}) + \boldsymbol{C} \times (\boldsymbol{A} \times \boldsymbol{B}) = 0$

(c) $\boldsymbol{A} \cdot (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{C}$

6) Two sides of a triangle are given by vectors $\boldsymbol{A} = [1, 2, 5], \boldsymbol{B} = [-3, 4, -2]$, determine the area of the triangle.

7) The edges of a parallelepiped are given by vectors $\boldsymbol{A} = [1+\frac{2}{\sqrt{2}}, 1-\frac{2}{\sqrt{2}}, 1], \boldsymbol{B} = [1-\frac{2}{\sqrt{2}}, 1+\frac{2}{\sqrt{2}}, -1], \boldsymbol{C} = [-1, 1, \sqrt{2}]$, determine the surface area and volume of the parallelepiped.

8) Vectors $\boldsymbol{A} = [1, 3, 1], \boldsymbol{B} = [2, -1, 3]$ define a plane. Find two vectors $\boldsymbol{x}$, $\boldsymbol{y}$ that lie in the same plane, such that $\boldsymbol{x} \cdot \boldsymbol{x} = 1$, $\boldsymbol{y} \cdot \boldsymbol{y} = 1$, and $\boldsymbol{x} \cdot \boldsymbol{y} = 0$.

9) A man stands in the field at a distance L from a straight road. A bus is moving along the road from the right. In what direction should the man run in order to cross the road in front and as far from the bus as possible? The speed of the bus is u, the speed of the man is v.