KLUB NAUKOWY FENIX

Problems 4

15 October, 2021

For reference use: W. Krysicki, L. Włodarski, Analiza matematyczna w zadaniach, PWN 1988, ISBN: 978-83-01-14295-7

1) Show that:

$$\text{a)}\ \left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}, \qquad \text{b)}\ [f(g(x))]' = f'(g(x))g'(x),$$

$$\text{c)}\ \left[f^{-1}(y)\right]' = \frac{1}{f'(x)} \quad \text{where} \quad y = f(x)$$

2) Find the derivative of a function with respect to x:

a) $\sqrt{x}$, b) $\sqrt[3]{x}$, c) $\sin x$, d) $\cos x$, e) $\sin^2 x$, f) $3xy + y^2 - x^2$,

g) $\sec(x)$, h) $\csc(x)$, i) $\operatorname{tg}(x)$, j) $\operatorname{ctg}(x)$, k) $\operatorname{arc\,tg}(x)$, l) $\dfrac{\sin x}{x}$,

m) $\operatorname{arc\,tg}(x)$, n) $\operatorname{arc\,cos}(x)$, o) $\operatorname{arc\,sin}(x)$, p) $\dfrac{1+x}{1-x}$, q) $\dfrac{a-x}{\sqrt{a^2-x^2}}$,

r) $\sqrt{1 + \operatorname{tg}\left(x + \dfrac{1}{x}\right)}$, s) $\cos^2\sqrt{\dfrac{1}{x}}$, t) $\dfrac{3\sin x}{\sqrt[3]{x^3+1}}$, u) $\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$,

v) $\exp(\alpha x)\,(a\sin x - \cos x)$ w) $\dfrac{1}{5}x^5 \operatorname{arc\,tg} x - \dfrac{1}{20}x^4 + \dfrac{1}{10}x^2 - \dfrac{1}{11}\log\left(1+x^2\right)$,

x) $\sin\left(xf(x)\right)$, y) $\sin(\sin x)$, z) $\sqrt[5]{x^2}$, α) $\operatorname{tg}^4\sqrt{x}$, β) x^x, γ) x^{x^x}.

2) Find a second derivative of a function with respect to x:

a) $\sin x$, b) x^2, c) $\exp(\alpha x)$, d) $\dfrac{x}{f(x)}$.

3) Find all maxima and minima of the function:

a) $\dfrac{a+x}{\left(x^2 - x_0^2\right)^2}$, b) $x\sqrt{4-x^2}$, c) $\sin x - \alpha x$, d) $2\operatorname{tg} x - \operatorname{tg}^2 x$.

5) A ball is thrown with velocity v up a hill whose inclination is β. Find the direction in which ball should be thrown to travel the farthest distance before landing on the hill.

6) A ball is thrown with velocity v at an angle $\alpha \in [-\pi/2, \pi/2]$ to the horizontal ground. Find a boundary of the space that can be reached by such ball before it touches the ground - an envelope of all possible trajectories.