KLUB NAUKOWY FENIX

# Problems 8

12 November, 2020

## Linear Differential Equations

1. Find first three non-zero terms of a Tailor series expansion around $x_0$ of the flowing functions:

   ($\alpha$) $\sin(x), \quad x_0 = 0$
   ($\beta$) $\cos(x), \quad x_0 = 0$
   ($\gamma$) $\sqrt{x}, \quad x_0 = 1$
   ($\delta$) $\ln(x), \quad x_0 = 1$
   ($\varepsilon$) $\sin(x), \quad x_0 = \pi/4$
   ($\zeta$) $\cos(x), \quad x_0 = \pi/4$

2. Find a general form of solution $y(x)$ to the following differential equations:

   ($\alpha$) $y''' - 27y = 0$
   ($\beta$) $y^{(6)} + y = 0$
   ($\gamma$) $y''' - 5y'' + 8y' = 0$
   ($\delta$) $y'' - y' + y = 0$
   ($\varepsilon$) $y'' - 4y' + y = 0$
   ($\zeta$) $y'' - 2y' + y = 0$
   ($\eta$) $y'' = a$
   ($\theta$) $y'' - y = e^x$

3. Find solution for $y(x)$ to the following differential equations:

   ($\alpha$) $y'' + 25y = 0, \quad y(0) = 0, \quad y'(0) = v_0$
   ($\beta$) $y'' + 16y = 0, \quad y(0) = y_0, \quad y'(0) = 0$
   ($\gamma$) $y'' - 4y = 0, \quad y(0) = 2, \quad y'(0) = -1$
   ($\delta$) $y'' = 1, \quad y(0) = 2, \quad y(2) = 1$
   ($\varepsilon$) $y''' = 1, \quad y(0) = 4, \quad y'(0) = -2, \quad y''(0) = 1$

4. By substituting $y(x) = f^2(x) - 2x_0$ find the solution to the differential equation:

   $$f(x)f'(x) + \tfrac{1}{2}f^2(x) = 2x_0$$

## Physics problems

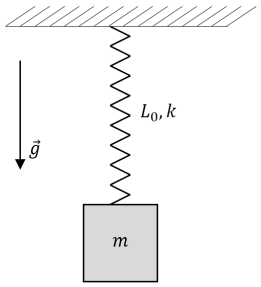


Figure 1

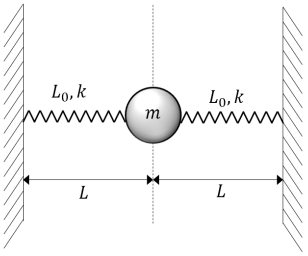


Figure 2

1. A block of mass $m$ is attached to a spring with free length $L_0$ and spring constant $k$ whose other end is attached to the celling (Figure 1). The system is subjected to gravitational field $g$. The block is held in the air such that the springs is not stretched or contracted. The block is released. Find the motion of the block assuming there is no friction in the system.

2. A ball is attached to two identical springs (with free length $L_0$ and spring constant $k$) that are also attached to the two opposite parallel walls (Figure 2). Find the equilibrium positions for the ball. Find the period of small oscillations of the ball when it is pushed out of equilibrium in the direction parallel to the walls. Consider cases when: $L < L_0$, $L > L_0$, and discuss the system behaviour when $L \approx L_0$.