Problems 9

19 November, 2021

Linear Differential Equations

1. Find a general form of solution $y(t)$ to the following differential equations:
   (a) $y''(t) + 2\beta\omega_0 y'(t) + \omega_0^2 y(t) = A_0 \sin(\omega t)$
   (b) $y''(t) + 2\beta\omega_0 y'(t) + \omega_0^2 y(t) = A_0 \cos(\omega t)$

   What is the difference between the solutions to a) and b)?

2. Find solution for $y(t)$ to the following differential equations:
   (a) $y''(t) + 2\beta\omega_0 y'(t) + \omega_0^2 y(t) = A_0 \cos(\omega t), \quad y(0) = y_0, \quad y'(0) = V_0$
   (b) $y''(t) = -1, \quad y(0) = y_0, \quad y'(0) = V_0$
   (c) $y''(t) - \lambda^2 y(t) = A_0 \cos(\omega t), \quad y(0) = y_0, \quad y'(0) = V_0$

3. Solve the system of equations for $f(x)$ and $g(x)$:

   (a)
   $$\begin{cases} f'(x) + 2g(x) = 0 \\ f(x) - 5g'(x) = 0 \end{cases}$$

   (b)
   $$\begin{cases} f'(x) - 7g(x) = \cos(x) \\ f(x) + 3g'(x) = 0 \end{cases}$$

4. Knowing that $y(t) = y_1 \cos(\omega t + \phi_1) + y_2 \cos(2\omega t + \phi_2)$ is a solution to the differential equation:
   $$y''(t) + 2\beta\omega_0 y'(t) + \omega_0^2 y(t) = A_1 \cos(\omega t) + A_2 \sin(2\omega t)$$
   and $\beta < 1$ find the dependence of y_1, y_2, ϕ_1, ϕ_2 on the system parameters: A_1, A_2, ω, ω_0, and β. Sketch the graph of the dependence of $\sqrt{y_1^2 + y_2^2}$ on the ω_0 assuming that: $A_1 = 1$, $A_2 = 0.5$, $\omega = 1$, and $\beta = 0.1$.

Physics problems

1. A ball placed at initial height $h = 0$ (close to the ground) is thrown with initial velocity V_0 at an angle α with respect to the ground. Assuming the ball is subjected to gravitational acceleration g find the trajectory of the ball and angle α_{MAX} at which the ball reaches the furthest distance before hitting the ground. Assume friction force F acting on the ball is opposite to the direction of the velocity with magnitude equal to $F(v) = \beta v$.

2. A block of mass M is hanging from the celling attached to a spring with spring constant k. Motion of the block is opposed by a friction force that is proportional to its velocity. An external force varying in time as $f(t) = f_0 \cos(\omega t)$ is applied to the block. Find and sketch the dependence of amplitude of block's vibrations as a function of:
   (a) Mass of the block m
   (b) Spring constant k