

Problems 3

14 October, 2022

For reference use: W. Krywicki, L. Włodarski, *Analiza matematyczna w zadaniach*, PWN 1988, ISBN: 978-83-01-14295-7

1) Show that:

$$\text{a) } \left[\frac{f(x)}{g(x)} \right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}, \quad \text{b) } [f(g(x))]' = f'(g(x))g'(x),$$

$$\text{c) } [f^{-1}(y)]' = \frac{1}{f'(x)} \quad \text{where } y = f(x)$$

2) Find the derivative of a function with respect to x :

$$\text{a) } \sqrt{x}, \quad \text{b) } \sqrt[3]{x}, \quad \text{c) } \sin x, \quad \text{d) } \cos x, \quad \text{e) } \sin^2 x, \quad \text{f) } 3xy + y^2 - x^2,$$

$$\text{g) } \sec(x), \quad \text{h) } \csc(x), \quad \text{i) } \operatorname{tg}(x), \quad \text{j) } \operatorname{ctg}(x), \quad \text{k) } \operatorname{arctg}(x), \quad \text{l) } \frac{\sin x}{x},$$

$$\text{m) } \operatorname{arctg}(x), \quad \text{n) } \operatorname{arccos}(x), \quad \text{o) } \operatorname{arcsin}(x), \quad \text{p) } \frac{1+x}{1-x}, \quad \text{q) } \frac{a-x}{\sqrt{a^2-x^2}},$$

$$\text{r) } \sqrt{1 + \operatorname{tg}\left(x + \frac{1}{x}\right)}, \quad \text{s) } \cos^2 \sqrt{\frac{1}{x}}, \quad \text{t) } \frac{3 \sin x}{\sqrt[3]{x^3 + 1}}, \quad \text{u) } \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}},$$

$$\text{v) } \exp(\alpha x) (a \sin x - \cos x) \quad \text{w) } \frac{1}{5}x^5 \operatorname{arctg} x - \frac{1}{20}x^4 + \frac{1}{10}x^2 - \frac{1}{11} \log(1+x^2),$$

$$\text{x) } \sin(xf(x)), \quad \text{y) } \sin(\sin x), \quad \text{z) } \sqrt[5]{x^2}, \quad \alpha) \operatorname{tg}^4 \sqrt{x}, \quad \beta) x^x, \quad \gamma) x^{x^x}.$$

2) Find a second derivative of a function with respect to x :

$$\text{a) } \sin x, \quad \text{b) } x^2, \quad \text{c) } \exp(\alpha x), \quad \text{d) } \frac{x}{f(x)}.$$

3) Find all maxima and minima of the function:

$$\text{a) } \frac{a+x}{(x^2-x_0^2)^2}, \quad \text{b) } x\sqrt{4-x^2}, \quad \text{c) } \sin x - \alpha x, \quad \text{d) } 2 \operatorname{tg} x - \operatorname{tg}^2 x.$$

5) A ball is thrown with velocity v up a hill whose inclination is β . Find the direction in which ball should be thrown to travel the farthest distance before landing on the hill.

6) A ball is thrown with velocity v at an angle $\alpha \in [-\pi/2, \pi/2]$ to the horizontal ground. Find a boundary of the space that can be reached by such ball before it touches the ground - an envelope of all possible trajectories.